Formulas

Chris Clark May 6, 2008

1 Introduction

This paper uses natural units. This will be fixed later. Note that while the Klein-Gordon probability density is not always positive, its integral over all space is constant with respect to time. I think we should be aiming to obtain $\rho_\phi = \rho_\psi$ where ϕ is the universal field for the case when there is one particle with wave function ψ.

2 Definition of Convolution

$$(A * B)(\mathbf{x}) = \int d^3x' A(\mathbf{x}')B(\mathbf{x} - \mathbf{x}')$$

3 Autoconvolution of $\tilde{\omega}$

$$(\tilde{\omega} * \tilde{\omega})(\mathbf{x}) = \int dx' \; \tilde{\omega}(\mathbf{x}')\tilde{\omega}(\mathbf{x} - \mathbf{x}')$$

$$= \int d^3x' \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}'}\omega(\mathbf{q}) \int \frac{d^3q'}{(2\pi)^3} e^{i\mathbf{q}'\cdot(\mathbf{x}-\mathbf{x}')}\omega(\mathbf{q}')$$

$$= \int \frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}\left(\int d^3x' \; e^{i(\mathbf{q}-\mathbf{q}')\cdot\mathbf{x}'}\right) e^{i\mathbf{q}'\cdot\mathbf{x}}\omega(\mathbf{q})\omega(\mathbf{q}')$$

$$= \int \frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}(2\pi)^3\delta^3(\mathbf{q} - \mathbf{q}')e^{i\mathbf{q}'\cdot\mathbf{x}}\omega(\mathbf{q})\omega(\mathbf{q}')$$

$$= \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}}\omega^2(\mathbf{q})$$

4 Section Two

$$(\nabla^2 - m^2)\phi$$

$$= (\nabla^2 - m^2)\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}}\tilde{\phi}(\mathbf{q})$$

$$= \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}}(-q^2 - m^2)\tilde{\phi}(\mathbf{q})$$

$$= -\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}}\omega^2(\mathbf{q})\tilde{\phi}(\mathbf{q})$$

$$= -\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}}\omega^2(\mathbf{q})\int d^3x' \; e^{-i\mathbf{q}\cdot\mathbf{x}'}\phi(\mathbf{x}')$$

$$= -\int d^3x' \; \phi(\mathbf{x}')\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{x}')}\omega^2(\mathbf{q})$$

$$= -\int d^3x' \; \phi(\mathbf{x}')(\tilde{\omega} * \tilde{\omega})(\mathbf{x} - \mathbf{x}')$$

$$= -(\phi * \tilde{\omega} * \tilde{\omega})(\mathbf{x})$$

5 Klein Gordon

Lets define σ_ψ by

$$\Phi_1^\psi[\phi] = \int d^3x'\ \sigma_\psi(\mathbf{x}')(\phi^* * \tilde{\omega})(\mathbf{x}')\Psi_0[\phi]$$

Then we can apply the method of Lagrange multipliers with the constraint

$$N = \int d^3x'\ \rho_\phi(\mathbf{x}') = \int d^3x'\ \frac{i}{2m}\left(\phi^*\dot{\phi} - \phi\dot{\phi}^*\right)$$

to maximize $\Psi_1^\psi[\phi]/\Psi_0[\phi]$

$$\lambda\frac{\delta}{\delta\phi^*(\mathbf{x})}\int d^3x'\ \frac{i}{2m}\left(\phi^*\dot{\phi} - \phi\dot{\phi}^*\right) = \frac{\delta}{\delta\phi^*(\mathbf{x})}\left(\frac{\Psi_1^\psi[\phi]}{\Psi_0[\phi]}\right)$$

$$\lambda\dot{\phi}(\mathbf{x}) = \frac{\delta}{\delta\phi^*(\mathbf{x})}\int d^3x'\ \sigma_\psi(\mathbf{x}')(\phi^* * \tilde{\omega})(\mathbf{x}')$$

$$\lambda\dot{\phi}(\mathbf{x}) = \int d^3x'\ \sigma_\psi(\mathbf{x}')\tilde{\omega}(\mathbf{x}' - \mathbf{x})$$

$$\lambda\dot{\phi}(\mathbf{x}) = \int d^3x'\ \sigma_\psi(\mathbf{x}')\tilde{\omega}(\mathbf{x} - \mathbf{x}')$$

$$\lambda\dot{\phi}(\mathbf{x}) = (\sigma_\psi * \tilde{\omega})(\mathbf{x})$$

If we take $\lambda = 1$ then

$$\dot{\phi} = \sigma_\psi * \tilde{\omega}$$

$$\ddot{\phi} = \dot{\sigma}_\psi * \tilde{\omega}$$

$$-\phi * \tilde{\omega} * \tilde{\omega} = \dot{\sigma}_\psi * \tilde{\omega}$$

If $\tilde{\omega}$ has a convolution inverse,

$$-\phi * \tilde{\omega} = \dot{\sigma}_\psi$$

$$-\dot{\phi} * \tilde{\omega} = \ddot{\sigma}_\psi$$

$$-\sigma_\psi * \tilde{\omega} * \tilde{\omega} = \ddot{\sigma}_\psi$$

$$(\nabla^2 - m^2)\sigma_\psi = \ddot{\sigma}_\psi$$

Therefore, σ_ψ must obey the Klein-Gordon equation. Notice that we have both $\dot{\phi} = \sigma_\psi * \tilde{\omega}$ and $\dot{\sigma}_\psi = -\phi * \tilde{\omega}$ so we cannot have $\phi = \sigma_\psi$.