Geometrical Optics

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Introduction

Geometrical optics is a formalism for analyzing simple optical systems. It models the optical phenomena of reflection and refraction, but not other phenomena such as diffraction and polarization.

A *point source* of light is a light source that emits light of equal intensity in all directions from a single point in space. When a point source is first turned on, light travels outward from this point at the speed of light, which is approximately 2.998×10^8 m/s. If we imagine the boundary that shows how far light has travelled from the point source, it will be a sphere whose radius grows at the speed of light. This boundary is known as the *wave front* of the light coming from the point source. When this wave front encounters an optical device such as a lens or mirror, it can become distorted and lose its spherical shape. In geometrical optics, these distortions can be tracked using *light rays*, drawn as lines or curves, that are perpendicular to the wave front at every step of the wave front's propagation.

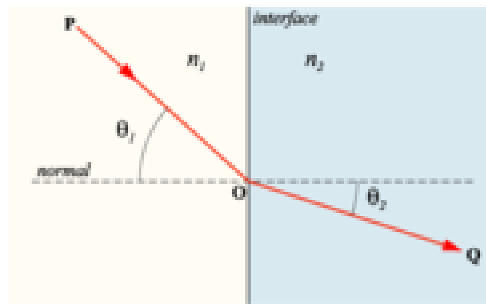
Properties of Light Ray Behavior

In order to use geometrical optics, we need to understand how light rays behave. Below are the three basic properties of light ray behavior.

(a) When light rays are in empty space, they just continue in a straight line.

(b) When a light ray strikes the surface of a mirror, reflection occurs. According to the law of reflection, the angle of the incident beam is equal to the angle of the reflected beam. By convention, these angles are measured with respect to the perpendicular to the surface of reflection.

(c) When a light ray encounters the surface of a lens upon entering or exiting, the ray is bent by a phenomenon called refraction. The law of refraction (also known as Snell's Law) says that the angle of the refracted beam θ_2 satisfies $n_1 \sin(\theta_1) = n_1 \sin(\theta_2)$, where n_1 is the index of refraction of the first medium, n_2 is the index of refraction of the second medium, and θ_1 is the angle of incidence, again measured with respect to the perpendicular to the lens' surface.



These three properties of light rays encode all the physics that is used in geometrical optics. Although these properties are technically sufficent to analyze any lens/mirror system, it is rather time consuming to use them directly. You would have to use a protractor every time a ray strikes the surface of a lens or mirror. Instead it is much better to utilize the special properties of ideal lenses and mirrors.

Ideal Lenses and Mirrors

The following properties are the definitions of the ideal lenses and mirrors. In these definitions, the parameter f is the *focal length*, which is a parameter that depends on the curvature and index of refraction. Also, the term *optical axis* refers to the axis of rotational symmetry of the device, perpendicular to the surface at the center point.

(a) An ideal convex (converging) lens sends rays parallel to the optical axis to a focal point a distance f on the other side of the lens.

(b) An ideal concave (diverging) lens causes rays parallel to the optical axis to diverge as if coming from a virtual focus a distance f in front of the lens.

(c) An ideal concave (converging) mirror sends rays parallel to the optical axis to a focal point a distance f in front of the mirror.

(d) An ideal convex mirror (diverging) causes rays parallel to the optical axis to diverge as if coming from a virtual focus a distance f behind the mirror.

Also, for any of these cases, the incoming and outgoing rays can be interchanged because lenses and mirrors are always symmetrical with respect to incoming and outgoing light rays.

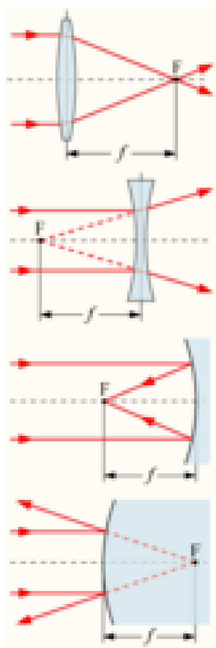
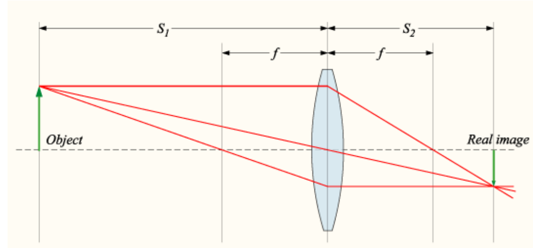


Image Formation

The most common use of geometrical optics is to find where and how images form. A *real image* is a configuration of light rays that will reproduce the appearance of an object if a screen is placed at its location. For example, if you have a light source and an object on one side of a convex lens, there will be some location on the other side where you can place a screen to see the object clearly in focus on the screen. If an image is in focus, then all light rays that originated from one point on the object must arrive at a single point in the image. If this is not the case, then rays from different parts of the object will mix and blend together on the screen, causing blurring. Using this fact that light rays must reconverge in order to form a real image, we can find the image location using a *ray diagram*.



How to Draw a Ray Diagram

(a) Draw the lens and its optical axis as a dotted line.

(b) Draw the object as an arrow starting somewhere on the optical axis pointing perpendicular to the axis.

(c) Draw a ray from the end of the arrow to the center of the lens and continuing straight through to the other side, or to the center of the mirror and reflecting off at the same angle.

(d) Draw a ray from the top of the arrow, parallel to the optical axis until it reaches the midpoint of the lens or the surface of the mirror, then bend this line toward the focus or directly away from the virtual focus according to the definition of the appropriate ideal mirror or lens.

(e) Find the intersection of these two rays and draw an arrow for the real image perpendicular to the optical axis, starting on the optical axis and extending to the point of intersection.

The first two steps are just to setup the problem. The third step is based on the thin-lens approximation. If the lens is not thin, then a ray heading toward the center of the lens will not emerge from the other side of the lens in the same direction. But if the lens is thin, the angle of refraction on the far side cancels out the angle from the near side since the situation is just inverted (air to glass, then glass to air, but otherwise symmetrical). The fourth step is justified based on the definition of ideal lenses and mirrors. The fifth step is a result of the definition of a real image.

Gaussian Lens Equation

Using the ray diagram for an ideal convex lens, we can find a relationship between the focal length and the distances from the lens to the object and image called the Gaussian lens equation,

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i},$$

where f is the focal length, d_o is the distance from the lens to the object, and d_i is the distance from the lens to the image.

To derive this equation, consider the figure of the ray diagram on the previous page. Let the height of the object be h_o and the height of the image be h_i. Then by similar triangles,

$$\frac{h_o}{d_o} = \frac{h_i}{d_i}$$

and

$$\frac{h_o}{f} = \frac{h_i}{d_i - f}.$$

Multiplying the second by $(d_i - f)/h_o$,

$$\frac{d_i - f}{f} = \frac{h_i}{h_o}$$

Using the first relation,

$$\frac{d_i - f}{f} = \frac{d_i}{d_o}$$

Dividing by d_i,

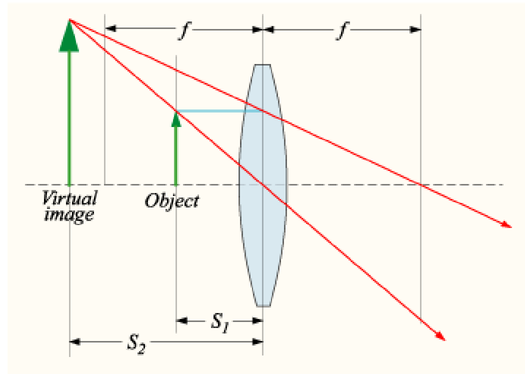
$$\frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o},$$

which gives the Gaussian Lens Equation upon rearrangement.

This equation can be applied to both convex and concave lenses if certain sign conventions are used. According to the convention, the focal length of concave (diverging) lenses is negative. This requires that either the image or object distance must be negative. This negative sign in the distance flips the side of the lens that the image or object is on, so if just one is negative, then both the image and object will be on the same side of the lens. When the image location is negative, the image is called a virtual image.

Virtual Images

There is a second type of image that is completely different from real images. A *virtual image* is a name given to the image that appears to us on the other side of a lens or mirror. For example, the image of yourself in a mirror is a virtual image, as is an enlarged view of text under a magnifying glass. Virtual images do not produce a focused configuration of light rays at the image location; it is just an apparent image, so it can never be projected on a screen at the image location. Our eyes are designed with a converging lens so that they can focus diverging rays because in the absence of any optical devices, rays from objects naturally diverge. This is why our eyes can focus the virtual image from the rays coming out of a diverging lens.



Combining Optical Devices

Some problems in geometrical optics involve a series of two or more lens and/or mirrors. For example, light rays may pass through two consecutive lens before forming an image on a screen. These problems can still be analyzed with the Gaussian Lens Equation if the following rules are obeyed.

- Trace through the path that light rays take through the system, starting from the light source.
- Every time the rays strike a device, compute the image location (whether real or virtual) with the Gaussian Lens Equation, and use the image at this location as a virtual object for the next device that the light rays will strike. Note that sometimes the image location will be on the far side of a barrier where no light can reach, but it still works because it is a mathematial trick.
- Be sure to use the proper sign conventions with negative distances for virtual images and objects.

In some cases you may have to leave some image locations as unknown variables and solve for them after writing down all the equations.

* Images courtesy of Wikipedia (user DrBob).