

# Formula Functions

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**Definition 0.1.** A **formula function** is an entire function obtained by a finite composition of

- (1) the parameter(s) of the function
- (2) constant numbers
- (3) functions from a finite set of elementary operators
- (4) previously defined formula functions.

## 1 Conventions

The set of elementary operators will be  $\{+, -, *, /, ^, !\}$ . Factorial can only be used on integers. Exponentiation will be single-valued, so any square root (or even root) will return the positive root.

To make functions that return logical truth values, we need some way of generating discontinuities. It is now a common convention that  $0^0 = 1$ , which provides a discontinuity in the function  $0^{|x|}$ . This function is similar to the Kronecker Delta function with one parameter fixed as 0 and it is very useful for logic functions.

## 2 Formula Functions

$$\begin{aligned} \text{abs}(x) &= \sqrt{x^2} & \text{gte}(x, y) &= \text{equ}(y, \text{min}(x, y)) \\ \text{avg}(x, y) &= \frac{1}{2}(x + y) & \text{lte}(x, y) &= \text{equ}(y, \text{max}(x, y)) \\ \text{max}(x, y) &= \text{avg}(x, y) + \frac{1}{2}\text{abs}(x - y) & \text{uni}(x) &= \text{gte}(x, 0) * \text{lte}(x, 1) \\ \text{min}(x, y) &= \text{avg}(x, y) - \frac{1}{2}\text{abs}(x - y) & \text{jag}(x) &= x * \text{uni}(x)(1 - \text{equ}(x, 1)) \\ \text{not}(x) &= 0^{\text{abs}(x)} & * \text{dec}(x) &= \sum_{n=-\infty}^{\infty} \text{jag}(x - n) \\ \text{equ}(x, y) &= \text{not}(x - y) & \text{int}(x) &= x - \text{dec}(x) \\ \text{sig}(x) &= \frac{x}{\text{abs}(x + \text{not}(x))} & \text{aux}(x, y) &= \frac{y}{x + 2y * \text{not}(x) + \text{not}(x) * \text{not}(y)} \\ \text{pos}(x) &= \text{equ}(x, \text{abs}(x)) & \text{div}(x, y) &= \text{equ}(\text{aux}(x, y), \text{int}(\text{aux}(x, y))) \\ \text{neg}(x) &= \text{equ}(x, -\text{abs}(x)) & \text{prm}(x) &= \text{div}(x, (x - 1)! + 1) \end{aligned}$$