Formula Functions

Chris Clark

March 19, 2004

Definition 0.1. A **formula function** is an entire function obtained by a finite composition of
(1) the parameter(s) of the function
(2) constant numbers
(3) functions from a finite set of elementary operators
(4) previously defined formula functions.

1 Conventions

The set of elementary operators will be {+,-,*,/,ˆ ,!}. Factorial can only be used on integers. Exponentiation will be single-valued, so any square root (or even root) will return the positive root.

To make functions that return logical truth values, we need some way of generating discontinuities. It is now a common convention that $0^0 = 1$, which provides a discontinuity in the function $0^{|x|}$. This function is similar to the Kronecker Delta function with one parameter fixed as 0 and it is very useful for logic functions.

2 Formula Functions

$abs(x) = \sqrt{x^2}$
$avg(x, y) = \frac{1}{2}(x + y)$
$max(x, y) = avg(x, y)+\frac{1}{2}abs(x-y)$
$min(x, y) = avg(x, y)-\frac{1}{2}abs(x-y)$
$not(x) = 0^{abs(x)}$
$equ(x, y) = not(x - y)$
$sig(x) = \frac{x}{abs(x+not(x))}$
$pos(x) = equ(x, abs(x))$
$neg(x) = equ(x, -abs(x))$
$gte(x, y) = equ(y, min(x, y))$
$lte(x, y) = equ(y, max(x, y))$
$uni(x) = gte(x, 0) * lte(x, 1)$
$jag(x) = x * uni(x)(1 - equ(x, 1))$
$*dec(x) = \Sigma_{n=-\infty}^{\infty} jag(x - n)$
$int(x) = x - dec(x)$
$aux(x, y) = \frac{y}{x+2y*not(x)+not(x)*not(y)}$
$div(x, y) = equ(aux(x, y), int(aux(x, y)))$
$prm(x) = div(x, (x - 1)! + 1)$