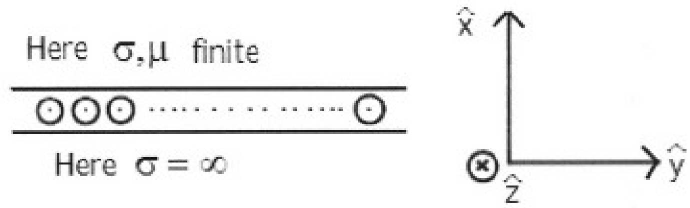
9. *Electricity and Magnetism* (Spring 2005)

An infinitely thin current sheet carrying a surface current $\lambda = \lambda_o \hat{z}\cos(\omega t)$ is sandwiched between a perfect conductor ($\sigma = \infty$) and a material having finite conductivity σ and magnetic permeability μ. The angular frequency ω is sufficiently low that magnetostatic conditions prevail. λ_o is a constant, $\hat{z}$ is a unit vector parallel to the interface located at $x = 0$, and t is the time.



(a) Find the appropriate partial differential equation that governs the behavior of the magnetic field **H** for $x > 0$ (above the current sheet). Do not solve.

(b) What is the appropriate boundary condition for **H** in this system?

(c) Find the magnetic field **H** at an arbitrary distance $x > 0$ at time t.

See Griffiths Ex 5.8

a. $\vec{\nabla}\times\vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t} = \lambda_o \hat{z}\cos(\omega t)\delta(x)$ since magnetostatics is the study of steady currents, so $\frac{\partial \vec{D}}{\partial t} = 0$.

$\Rightarrow \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ H_x & H_y & H_z \end{vmatrix}_z = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \lambda_o \cos(\omega t)\,\delta(x)$

b. $H_\perp^2 - H_\perp^1 = 0 \Rightarrow H_x(0,y,z) = 0$ since it must be zero inside a perfect conductor, so it is also zero on the other side.

$\vec{H}_\parallel^2 - \vec{H}_\parallel^1 = \vec{K}_f \times \hat{n} \Rightarrow \vec{H}_\parallel(0,y,z) = \vec{\lambda}\,\hat{y}$

Combining these two we get $\vec{H}(0,y,z) = \lambda_o \cos(\omega t)\,\hat{y}$

c. The problem is totally symmetric in the y direction.

so $\frac{\partial H_x}{\partial y} = 0 \Rightarrow \frac{\partial H_y}{\partial x} = \lambda_o \cos(\omega t)\,\delta(x)$

$\Rightarrow H_y(x,y,z,t) = \int_0^x \lambda_o \cos(\omega t)\delta(x')dx' + C$

$= \lambda_o \cos(\omega t) + C$

Now $C=0$ because of the B.C., so $H_y(x,y,z) = \lambda_o \cos(\omega t)$

The field can't have a z-component because the field must be perpendicular to the current by the Biot-Savart law. It also can't have an x-component because contributions from $-y$ cancel those from y.

Therefore $\vec{H}(x,y,z,t) = \lambda_o \cos(\omega t)\,\hat{y}$