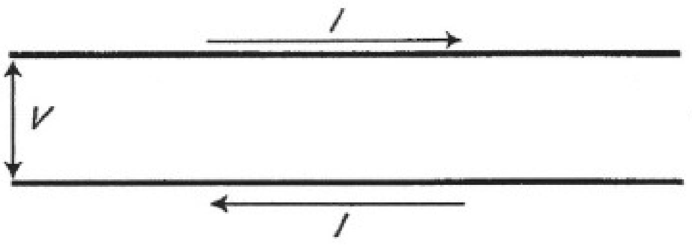
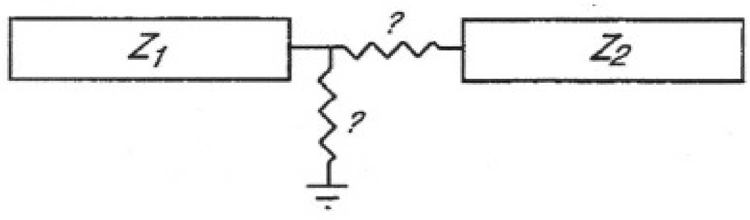
9. *Electricity and Magnetism* (Fall 2003)



(a) A two-wire transmission line has inductance L and capacitance C per unit length (and no resistance). Show that the impedance of this transmission line $Z = V/I$ is real and equal to $\sqrt{L/C}$ (Note: Assume AC signals are transmitted on the line, $I = I_o \exp(ikx - i\omega t)$).

(b) Two long transmission lines are connected together. The first has impedance Z_1 and the second has impedance $Z_2 \neq Z_1$. A wave $V_i \exp(ikx - i\omega t)$ travels on the first transmission line and encounters the second. What are the relative amplitudes of the reflected and transmitted waves ($V_r/V_i, V_t/V_i$)?



(c) Reflection due to impedance mismatch between two transmission lines can be eliminated through adding series or parallel resistance between the lines. For the transmission lines in (b), how would you connect a resistor (and what is its values) in order to match the impedances and eliminate the reflected wave? (Consider both $Z_1 > Z_2$ and $Z_1 < Z_2$.)

a. $Q = CV \Rightarrow I = \frac{dQ}{dt} = C\frac{dV}{dt} = C\frac{d}{dt}\left(-\frac{d\Phi_B}{dt}\right) = -C\frac{d^2}{dt^2}(LI)$

$= -CL(-i\omega)^2 I \Rightarrow CL\omega^2 = -1 \Rightarrow \omega = \pm i\sqrt{\frac{1}{CL}}$

$\Rightarrow V = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt} = i\omega L I = \pm\sqrt{\frac{L}{C}}\, I \Rightarrow Z = \frac{V}{I} = \sqrt{\frac{L}{C}}$

b. Recall $Z = \sqrt{\frac{\mu}{\epsilon}}$ and $v = \frac{c}{n}$ so $n = \frac{c}{v} = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}} \cong \frac{Z_0}{Z}$ since $\mu \cong \mu_0$

Then for normal incidence $\frac{E_0^r}{E_0} = \frac{n'-n}{n'+n}$ and $\frac{E_0^t}{E_0} = \frac{2n}{n'+n}$

$\Rightarrow \frac{V_r}{V_i} = \frac{n_2 - n_1}{n_2 + n_1} = \frac{\frac{1}{Z_2} - \frac{1}{Z_1}}{\frac{1}{Z_2} + \frac{1}{Z_1}} = \frac{Z_1 - Z_2}{Z_1 + Z_2}$, $\frac{V_t}{V_i} = \frac{2n_1}{n_2 + n_1} = \frac{2\frac{1}{Z_1}}{\frac{1}{Z_2} + \frac{1}{Z_1}} = \frac{2Z_2}{Z_1 + Z_2}$

c. We want to make the total impedance after Z_1 equal to Z_1, and since Z is real this just means equating the resistances, where Z_1 and Z_2 are themselves resistances.

Case $Z_1 > Z_2$: Series resistor $R = Z_1 - Z_2$

Case $Z_1 < Z_2$: Parallel resistor $\frac{1}{R} + \frac{1}{Z_2} = \frac{1}{Z_1} \Rightarrow R = \frac{Z_1 Z_2}{Z_2 - Z_1}$