2. *Quantum Mechanics* (Fall 2003)

A free particle of mass m, travelling with momentum p parallel to the z-axis, scatters off the potential

$$V = V_0 \left[\delta(\mathbf{r} - a\hat{\mathbf{z}}) - \delta(\mathbf{r} + a\hat{\mathbf{z}})\right].$$

Compute the differential cross section, $d\sigma/d\Omega$ in the Born approximation.

$$\frac{d\sigma}{d\Omega} = |f^{(1)}(\theta,\phi)|^2 \quad \text{where} \quad f^{(1)}(\vec{k}',\vec{k}) = -\frac{1}{4\pi}\frac{2m}{\hbar^2}\int e^{i(\vec{k}-\vec{k}')\cdot\vec{x}'} V(\vec{x}')\,d^3x'$$

$$\vec{k} = k\hat{z} \Rightarrow f^{(1)}(\vec{k}',\vec{k}) = -\frac{1}{4\pi}\frac{2m}{\hbar^2}\int e^{ikz' - i\vec{k}'\cdot\vec{x}'} V_0\left[\delta^3(\vec{x}'-a\hat{z}) - \delta^3(\vec{x}'+a\hat{z})\right]d^3x'$$

$$= -\frac{1}{4\pi}\frac{2m}{\hbar^2}V_0\left[e^{ika - ik_z'a} - e^{-ika + ik_z'a}\right]$$

$$= -\frac{1}{4\pi}\frac{2m}{\hbar^2}V_0\left[2i\sin((k-k_z')a)\right]$$

Now $k_z' = k'\cos(\theta) = k\cos(\theta)$ since $k' = k$ by conservation of energy assuming the scattering body is much larger.

$$f^{(1)}(\theta,\phi) = -\frac{imV_0}{\pi\hbar^2}\sin(ak(1-\cos(\theta)))$$

$$\frac{d\sigma}{d\Omega} = |f^{(1)}(\theta,\phi)|^2 = \frac{m^2V_0^2}{\pi^2\hbar^4}\sin^2(ak(1-\cos(\theta)))$$

$$= \frac{m^2V_0^2}{\pi^2\hbar^4}\sin^2(2ak\sin^2(\theta/2))$$