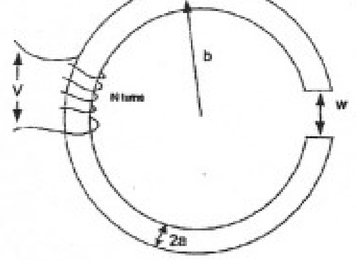
9. *Electricity and Magnetism* (Fall 2002)

A D.C. electromagnet is constructed from a cylindrical soft-iron bar with radius a. The relative magnetic permeability of the iron is μ. The bar is bent into a C-shape as shown below with radius b. The width of the small gap is w. The magnet is energized by winding a coil of copper wire N turns tightly around the bar and connecting the coil to a D.C. power supply with voltage V. The copper wire has resistivity ρ, and radius r_{wire}. Assume $r_{wire} \ll a \ll b$ and ignore fringe-field effects.

$$\Phi = LI \Rightarrow \mathcal{E} = -L\frac{dI}{dt} \Rightarrow V = V_0 + \mathcal{E} = V_0 - L\frac{dI}{dt} = IR$$



$$\frac{dI}{dt} = \frac{V_0}{L} - \frac{R}{L}I$$

$$\frac{dI}{\frac{V_0}{L} - \frac{R}{L}I} = dt$$

$$-\frac{L}{R}\ln\left(\frac{V_0}{L} - \frac{R}{L}I\right) = t + c$$

$$\frac{V_0}{L} - \frac{R}{L}I = Ae^{-\frac{R}{L}t}$$

$$I = \frac{V_0}{R} - A'e^{-t/(L/R)}$$

$$= \frac{V_0}{R} - A'e^{-t/\tau} \Rightarrow \tau = \frac{L}{R}$$

(a) What is the steady-state value of the magnetic field B in the gap?

(b) What is the time constant governing the response of the current in the coil when the voltage is turned on? (Assume μ is constant.)

a. Use Ampere's Law in terms of $\vec{H}$ so we can neglect magnetization currents.

Let C be a closed loop through the center of the ring.

$$\oint_C \vec{H}\cdot d\vec{l} = I_{enc} = NI$$

$$(2\pi b - w)H_{Fe} + w\,H_{gap} = NI$$ since w is small, $w \cong$ arclength

Now $\vec{B}$ is constant all the way around C (including the gap) since the normal component of $\vec{B}$ is continuous across boundaries.

$$H_{Fe} = \frac{1}{\mu}B \qquad H_{gap} = \frac{1}{\mu_0}B$$

$$\Rightarrow (2\pi b - w)\frac{1}{\mu}B + w\frac{1}{\mu_0}B = NI$$

$$\Rightarrow B = \frac{NI}{\frac{1}{\mu}(2\pi b - w) + \frac{1}{\mu_0}w}$$

Now $V = IR$ and $R = \rho\frac{L}{A} \cong \rho\frac{2\pi a N}{\pi r_w^2} = \frac{2\rho a N}{r_w^2}$

$$\Rightarrow B = \frac{V r_w^2}{(2\rho a)\left[\frac{1}{\mu}(2\pi b - w) + \frac{1}{\mu_0}w\right]}$$

b. $\tau = \frac{L}{R}$ where L is defined by $\Phi = LI$

$$\Phi = \int_{coil}\vec{B}\cdot d\vec{a} = N\int_{loop}\vec{B}\cdot d\vec{a} = N(\pi a^2)B$$

since B is the same throughout the ring and gap.

$$\tau = \frac{L}{R} = \frac{\Phi}{IR} = \frac{N(\pi a^2)B}{V} = \frac{N\pi r_w^2 a}{(2\rho)\left[\frac{1}{\mu}(2\pi b - w) + \frac{1}{\mu_0}w\right]}$$