

Force:	$\mathbf{F} = \frac{d\mathbf{p}}{dt}$	Torque on Electric Dipole:	$\mathbf{N} = \mathbf{p} \times \mathbf{E}$
Angular Momentum:	$\mathbf{L} = \mathbf{r} \times \mathbf{p}$	Torque on Magnetic Dipole:	$\mathbf{N} = \mathbf{m} \times \mathbf{B}$
Torque:	$\mathbf{N} = \frac{d\mathbf{L}}{dt} = \mathbf{r} \times \mathbf{F}$	Electromotive Force:	$\varepsilon = -\frac{d\Phi_B}{dt}$
Work (Energy):	$W = \int \mathbf{F} \cdot d\mathbf{l}$	Bound Charge:	$\rho_b = -\nabla \cdot \mathbf{P} \quad \sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$
Potential Energy:	$\mathbf{F} = -\nabla U$	Bound Current:	$\mathbf{J}_b = \nabla \times \mathbf{M} \quad \mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$
Total Energy:	$E = T + U$	Gradient of Potential:	$\nabla \left(\frac{1}{r} \right) = \frac{\hat{\mathbf{r}}}{r^2}$
Pressure:	$p = F/A$	Divergence of Electric Field:	$\nabla \cdot \left(\frac{\hat{\mathbf{r}}}{r^2} \right) = 4\pi\delta^3(\mathbf{r})$
Power:	$P = \frac{dE}{dt} = \mathbf{F} \cdot \mathbf{v}$	Spherical Method of Images: $x' = \left(\frac{a}{x}\right)a \quad q' = -\left(\frac{a}{x}\right)q$	
Electric Field:	$\mathbf{F} = q\mathbf{E}$	Electric Field:	$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$
Current:	$\mathbf{J} = \rho\mathbf{v}$	Field Energy:	$U = \frac{1}{2} \int \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) d^3x$
Electric Potential:	$V = -\int \mathbf{E} \cdot d\mathbf{l}$	Electric Field Outside a Conductor:	$\mathbf{E} = \frac{\sigma}{\epsilon_0} \hat{\mathbf{n}}$
Magnetic Flux:	$\Phi_B = \int_S \mathbf{B} \cdot d\mathbf{a}$	Magnetic Field of a Solenoid:	$\mathbf{B} = \mu_0 n I \hat{\mathbf{z}}$
Inductance:	$\Phi_B = LI$	Magnetic Field of a Wire:	$\mathbf{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$
Resistance:	$V = IR$	Resistance:	$R = \rho L/A$
Capacitance:	$Q = CV$	Circuit Impedance:	$Z_L = i\omega L \quad Z_C = \frac{1}{i\omega C}$
Conductivity/Resistivity:	$\mathbf{J} = \sigma\mathbf{E} = \frac{1}{\rho}\mathbf{E}$	Impedance of a Medium:	$Z = \sqrt{\frac{\mu}{\epsilon}}$
Poynting Vector:	$\mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B})$	Poisson's Equation:	$\nabla^2 V = -\frac{\rho}{\epsilon_0} \quad \nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$
Vector Potential:	$\mathbf{B} = \nabla \times \mathbf{A}$	Electric Potential:	$V(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{x}')}{ \mathbf{x}-\mathbf{x}' } d^3x'$
Electric Auxiliary Field:	$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$	Vector Potential:	$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}')}{ \mathbf{x}-\mathbf{x}' } d^3x'$
Magnetic Auxiliary Field:	$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$		
Impedance:	$Z = \frac{V}{I} = R + iX$		
Phase Velocity:	$v_p = \frac{\omega}{k}$	Snell's Law:	$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$
Group Velocity:	$v_g = \frac{d\omega}{dk}$	Plasma Dispersion Relation:	$k^2 c^2 \cong \omega^2 - \omega_p^2$

Hooke's Law:	$\mathbf{F} = -k\mathbf{x}$	Plasma Susceptibility:	$\frac{\epsilon(\omega)}{\epsilon_0} \cong 1 - \frac{\omega_p^2}{\omega^2}$
Coulomb's Law:	$\mathbf{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$	Plasma Frequency:	$\omega_p^2 = \frac{ne^2}{\epsilon_0 m}$
Biot-Savart Law:	$\mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{l}' \times \hat{\mathbf{r}}}{r^2}$	Skin Depth:	$\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$
Lorentz Force Law:	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$	Speed of light in medium:	$v = c/n = 1/\sqrt{\mu\epsilon}$
Gauss' Law:	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$	Fields of a plane wave:	$E = cB$
Faraday's Law:	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	Larmor Formula:	$P = \frac{\mu_0}{6\pi c} q^2 a^2$
Magnetic Gauss' Law:	$\nabla \cdot \mathbf{B} = 0$	Complex Poynting Vector:	$\tilde{\mathbf{S}} = \mathbf{E} \times \tilde{\mathbf{H}}^*$
Maxwell-Ampere Law:	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	Power per solid angle:	$\langle \frac{dP}{d\Omega} \rangle = \frac{1}{2} \text{Re}[r^2 \hat{\mathbf{n}} \cdot \tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*]$
Continuity Equation:	$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$		

Perpendicular Electric B.C.:	$\epsilon_1 E_1^\perp - \epsilon_2 E_2^\perp = \sigma_f$	Relativistic Energy:	$E^2 = p^2 c^2 + m^2 c^4$
Parallel Electric B.C.:	$\mathbf{E}_1^\parallel - \mathbf{E}_2^\parallel = 0$	Relativistic Energy for Massive Particle:	$E = \gamma mc^2$
Perpendicular Magnetic B.C.:	$B_1^\perp - B_2^\perp = 0$	Relativistic Momentum:	$\mathbf{p} = \gamma m \mathbf{u}$
Parallel Magnetic B.C.:	$\frac{1}{\mu_1} \mathbf{B}_1^\parallel - \frac{1}{\mu_2} \mathbf{B}_2^\parallel = \mathbf{K}_f \times \hat{\mathbf{n}}$	Relativity Notation:	$\gamma = 1/\sqrt{1-\beta^2} \quad \beta = \mathbf{v}/c$

Potential Energy of Point Charge:	$U = qV$	Field Trans. (cgs): $\mathbf{E}' = \gamma(\mathbf{E} + \beta \times \mathbf{B}) - \frac{\gamma^2}{\gamma+1} \beta(\beta \cdot \mathbf{E})$	
Dipole Moment:	$\mathbf{p} = \int \mathbf{x}' \rho(\mathbf{x}') d^3x' \rightarrow q\mathbf{d}$	Field Trans. (cgs): $\mathbf{B}' = \gamma(\mathbf{B} - \beta \times \mathbf{E}) - \frac{\gamma^2}{\gamma+1} \beta(\beta \cdot \mathbf{B})$	
Magnetic Moment:	$\mathbf{m} = \frac{1}{2} \int \mathbf{x}' \times \mathbf{J}(\mathbf{x}') d^3x' \rightarrow I\mathbf{a}$	Green's Functions:	$\nabla^2 G(\mathbf{x}, \mathbf{x}') = \delta^3(\mathbf{x} - \mathbf{x}')$
Linear Medium D Field:	$\mathbf{D} = \epsilon\mathbf{E}$	Fundamental Solution:	$G(\mathbf{x}, \mathbf{x}') = -\frac{1}{4\pi} \frac{1}{ \mathbf{x}-\mathbf{x}' }$
Linear Medium H Field:	$\mathbf{H} = \frac{1}{\mu}\mathbf{B}$	Azimuthal:	$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos(\theta))$
Polarization:	$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$	Divergence Theorem:	$\int_V \nabla \cdot \mathbf{A} d^3x = \int_S \mathbf{A} \cdot d\mathbf{a}$
Magnetization:	$\mathbf{M} = \chi_m \mathbf{H}$	Stokes' Theorem:	$\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint_C \mathbf{A} \cdot d\mathbf{l}$
Electrostatic Pressure:	$p = \sigma E_{avg}$	Spherical Gradient:	$\nabla = \frac{\partial}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{\theta} + \frac{1}{r \sin(\theta)} \frac{\partial}{\partial \phi} \hat{\phi}$
Electric Dipole Potential:	$V_{dip} = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}$	Cylindrical Gradient:	$\nabla = \frac{\partial}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial}{\partial \phi} \hat{\phi} + \frac{\partial}{\partial z} \hat{\mathbf{z}}$
Electric Dipole Energy:	$U = -\mathbf{p} \cdot \mathbf{E}$	Cylindrical Laplacian:	$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}$
Magnetic Dipole Energy:	$U = -\mathbf{m} \cdot \mathbf{B}$		

Spherical Laplacian:	$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta} \left(\sin(\theta) \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2}{\partial \phi^2}$
Green's Functions Potential:	$V(\mathbf{x}) = -4\pi \int_V G(\mathbf{x}, \mathbf{x}') \rho(\mathbf{x}') d^3x' + \oint_S \left(V(\mathbf{x}') \frac{\partial G(\mathbf{x}, \mathbf{x}')}{\partial n'} - G(\mathbf{x}, \mathbf{x}') \frac{\partial V(\mathbf{x}')}{\partial n'} \right) da'$
Spherical Harmonic Expansion:	$\frac{1}{ \mathbf{x}-\mathbf{x}' } = \sum_{l=0}^{\infty} \sum_{m=-l}^l \frac{r_{<}^l}{r_{>}^{l+1}} P_l(\cos(\gamma)) = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^l \frac{1}{2l+1} \frac{r_{<}^l}{r_{>}^{l+1}} Y_{lm}^*(\theta', \phi') Y_{lm}(\theta, \phi)$
Multipole Expansion:	$V(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{x}')}{ \mathbf{x}-\mathbf{x}' } d^3x' = \frac{1}{\epsilon_0} \sum_{l=0}^{\infty} \sum_{m=-l}^l \frac{1}{2l+1} \left[\int Y_{lm}^*(\theta', \phi') r'^l \rho(\mathbf{x}') d^3x' \right] \frac{Y_{lm}(\theta, \phi)}{r^{l+1}}$
BAC-BAC (Back to Back) Vector Identity:	$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - (\mathbf{B} \cdot \mathbf{A})\mathbf{C}$

Fundamentals

Probability Postulate:	$P(\lambda) = \langle \lambda \psi \rangle ^2$
Shrodinger's Equation:	$-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = i\hbar \frac{\partial}{\partial t} \psi$
Momentum Operator:	$\mathbf{p} = -i\hbar \nabla$
de Broglie Relation:	$p = \frac{h}{\lambda} = \hbar k$
Expectation Value:	$\langle A \rangle = \langle \psi A \psi \rangle$
Definition of Unitary:	$U^\dagger = U^{-1}$
Stationary States:	$ \psi(t)\rangle = e^{-iHt/\hbar} \psi(0)\rangle$
Canonical Commutation Relation:	$[x_i, p_j] = i\hbar \delta_{ij}$
Jacobi Identity:	$[A, BC] = B[A, C] - [B, A]C$
Identity Operator:	$I = \sum_i \phi_i\rangle \langle \phi_i $
Hermitian Conjugate:	$\langle \psi_1 A^\dagger \psi_2 \rangle = \langle \psi_2 A \psi_1 \rangle^*$
Definition of Hermitian:	$A^\dagger = A$
Hermitian Conjugate of Product:	$(AB)^\dagger = B^\dagger A^\dagger$
Uncertainty Principle:	$\sigma_A^2 \sigma_B^2 \geq -\frac{1}{4} \langle \psi [A, B] \psi \rangle^2$
Variational Method:	$\langle \psi H \psi \rangle \geq E_0 \quad \forall \psi\rangle$
Expectation Evolution:	$\frac{d\langle Q \rangle}{dt} = \frac{i}{\hbar} \langle [H, Q] \rangle + \langle \frac{\partial Q}{\partial t} \rangle$
Generalized Coordinates:	$p \equiv \frac{\partial L}{\partial \dot{q}} \quad \dot{p} = -\frac{\partial H}{\partial q} \quad \dot{q} = \frac{\partial H}{\partial p}$

Mathematics

Variance:	$\sigma_x^2 \equiv \langle (x - \langle x \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$
Euler's Equation:	$e^{ix} = \cos(x) + i \sin(x)$
Gamma Integral:	$\int_0^\infty x^n e^{-x} dx = \Gamma(n+1)$
Gaussian Integral:	$\int_0^\infty x^n e^{-x^2} dx = \frac{1}{2} \Gamma(\frac{n+1}{2})$
Gamma Properties:	$\Gamma(n+1) = n\Gamma(n), \quad \Gamma(\frac{1}{2}) = \sqrt{\pi}$
Exponential Approximation:	$e^x \cong 1 + x \quad (x \ll 1)$
Square Root Approximation:	$\sqrt{1+x} \cong 1 + \frac{x}{2} \quad (x \ll 1)$

Simple Harmonic Oscillator

Lowering Operator:	$a = \frac{1}{\sqrt{2m\omega\hbar}}(m\omega x + ip)$
Hamiltonian:	$H_{SHO} = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = (a^\dagger a + \frac{1}{2})\hbar\omega$
Energy Levels:	$E_n = (n + \frac{1}{2})\hbar\omega$
Position Dimensions:	$x_o = \sqrt{\frac{\hbar}{2m\omega}}$
Momentum Dimensions:	$x_o p_o = \hbar/2$
Position Operator:	$x = x_o(a^\dagger + a)$
Momentum Operator:	$p = ip_o(a^\dagger - a)$
Ladder Operators:	$[a, a^\dagger] = 1$
Raising:	$a^\dagger \psi_n\rangle = \sqrt{n+1} \psi_{n+1}\rangle$
Lowering:	$a \psi_n\rangle = \sqrt{n} \psi_{n-1}\rangle$
Number Operator:	$N \psi_n\rangle = a^\dagger a \psi_n\rangle = n \psi_n\rangle$
Ground State:	$a \psi_0\rangle = 0$

Selection Rules: $\langle \alpha' j' m' | T_q^k | \alpha j m \rangle = 0$ unless $m' = m + q$ and $|j - k| \leq j' \leq j + k$

Perturbation Theory: $\Delta_n = E_n - E_n^0 = \langle \psi_n^0 | H' | \psi_n^0 \rangle - \sum_{m \neq n} \frac{|\langle \psi_m^0 | H' | \psi_n^0 \rangle|^2}{E_m^0 - E_n^0} + \dots$

Time-Dependent Perturbation Theory: $\langle \psi_f | \psi(t) \rangle \cong \delta_{fi} - \frac{i}{\hbar} \int_0^t \langle \phi_f | H^1(t') | \phi_i \rangle e^{i\omega_{fi}t'} dt'$

WKB Approximation for Semi-Infinite Potential: $\int_0^{x_2} \sqrt{2m(E_n - V(x))} dx = (n - \frac{1}{4})\pi\hbar \quad (n \in \mathbb{Z}^+)$

Baker-Hausdorff Lemma: $e^{iB} A e^{-iB} = A + i[B, A] - \frac{1}{2}[B, [B, A]] + \dots + \frac{i^n}{n!}[B, [B, [B, \dots, [B, A]]]] + \dots$

Angular Momentum

L_z Operator:	$L_z = -i\hbar \frac{\partial}{\partial z}$
Definition of \mathbf{J} :	$\mathbf{J} = \mathbf{L} + \mathbf{s}$
J_z Eigenvalue:	$J_z j m\rangle = \hbar m j m\rangle$
J^2 Eigenvalue:	$\mathbf{J}^2 j m\rangle = \hbar^2 j(j+1) j m\rangle$
Angular Momentum Operators:	$(\bar{J}_i)_{jk} = -i\epsilon_{ijk}$
Commutator:	$[\bar{J}_i, \bar{J}_j] = i \sum_k \epsilon_{ijk} \bar{J}_k$
Ladder Operators:	$J_\pm = J_x \pm iJ_y$
Ladder:	$J_\pm j, m\rangle = \hbar \sqrt{j(j+1) - m(m \pm 1)} j, m \pm 1\rangle$

Spin and Magnetism

Pauli Matrices:	$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Spin Singlet:	$ 0, 0\rangle = \frac{1}{\sqrt{2}}(+-\rangle - -+\rangle)$
Spin Triplet ($m=0$):	$ 1, 0\rangle = \frac{1}{\sqrt{2}}(+-\rangle + -+\rangle)$
Spin Triplet ($m \neq 0$):	$ 1, 1\rangle = ++\rangle \quad 1, -1\rangle = --\rangle$
Magnetism:	$H_{mag} = -\boldsymbol{\mu} \cdot \mathbf{B} = \mu_B (\mathbf{L} \cdot \mathbf{B} + \boldsymbol{\sigma} \cdot \mathbf{B})$
Electron Magnetic Moment:	$\boldsymbol{\mu} = -\mu_B \mathbf{L}$
Bohr Magneton:	$\mu_B = \frac{e\hbar}{2m}$
Spin Half Polarization:	$\frac{d\mathbf{P}}{dt} = \mathbf{H} \times \mathbf{P}$
Spin-Orbit Hamiltonian:	$H_{LS} = \frac{\alpha}{2m^2} \frac{1}{r^3} \mathbf{L} \cdot \mathbf{s}$

Atoms

Fine Structure Constant:	$ke^2 = \alpha\hbar c$
Bohr Radius:	$a = \frac{\hbar}{m\alpha c}$
Reduced Mass:	$\mu = \frac{m_1 m_2}{m_1 + m_2}$
Hydrogen-like Atom Hamiltonian:	$H = \frac{p^2}{2\mu} - \frac{Ze^2}{r}$
Hydrogen Energies:	$E_n = -\frac{mk^2 Z^2 e^4}{2\hbar^2 n^2} = -\frac{Z^2 \alpha^2}{2n^2} mc^2$
Virial Theorem:	$2\langle \psi T \psi \rangle = \langle \psi r \frac{\partial V}{\partial r} \psi \rangle$
Coulomb Virial Theorem:	$\langle E \rangle = -\langle T \rangle$
Hydrogen Ground State:	$\psi_1(r, \theta, \phi) = \frac{a^{-3/2}}{\sqrt{\pi}} e^{-r/a}$

Scattering

Wavefunction:	$\psi(r) = \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}} + f(\theta, \phi) \frac{e^{ikr}}{r} \right]$
Cross Section:	$\sigma_{tot} = \int \frac{d\sigma}{d\Omega} d\Omega = \int f(\theta, \phi) ^2 d\Omega$
Amp.:	$f = \frac{-(2\pi)^{3/2}}{4\pi} \frac{2m}{\hbar^2} \int e^{-i\mathbf{k}'\cdot\mathbf{x}'} V(\mathbf{x}') \psi(\mathbf{x}') d^3x'$
Born:	$f^{(1)}(\mathbf{k}', \mathbf{k}) = -\frac{1}{4\pi} \frac{2m}{\hbar^2} \int V(\mathbf{x}') e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}'} d^3x'$
Sym. Born:	$f^{(1)}(\theta, \phi) = -\frac{2m}{\hbar^2} \frac{1}{q} \int_0^\infty r V(r) \sin(qr) dr$
Momentum Transfer:	$q \equiv \mathbf{k} - \mathbf{k}' = 2k \sin(\theta_{sc}/2)$
Optical Theorem:	$\sigma_{tot} = \frac{4\pi}{k} \text{Im}(f(0))$

Fundamentals

Fundamental Relation:	$dE = TdS - pdV + \mu dN$
Heat Definition:	$Q \equiv \Delta E + W$
Entropy Definition:	$S \equiv k \ln(\Omega)$
Beta Definition:	$\beta \equiv \frac{\partial \Omega}{\partial E}$
Temperature Definition:	$T \equiv \frac{1}{k\beta}$
Boltzmann Factor:	$P(\epsilon) \propto e^{-\beta\epsilon}$
Ideal Gas Law:	$pV = NkT$
Ideal Gas Energy:	$E = \frac{3}{2}NkT$
Pressure:	$p = -\left(\frac{\partial E}{\partial V}\right)_S$
Heat Capacity Definition:	$C_y \equiv \left(\frac{dQ}{dT}\right)_y$
Heat Capacity (Volume):	$C_V = \left(\frac{\partial E}{\partial T}\right)_V = T \left(\frac{\partial S}{\partial T}\right)_V$
Chemical Potential:	$\mu \equiv -T \left(\frac{\partial S}{\partial N}\right)_{EVM} = \left(\frac{\partial E}{\partial N}\right)_{SVN}$
Magnetic Susceptibility Definition:	$M_0 = \chi H$
Magnetization:	$\bar{M}_0 = N_0 \bar{\mu}_H = \sum_r \mu_{H_r} n_r$
Magnetization Relation:	$M_0 = M/V$
Magnetic Moment:	$\mu = g\mu_B \mathbf{s}/\hbar$
Efficiency:	$\eta = \frac{T_1 - T_2}{T_1}$
Equipartition Theorem:	$\bar{\epsilon}_i = \frac{1}{2}kT$
Sterling Approximation:	$\ln(N!) \cong N \ln(N) - N$

Partition Functions

Partition Function:	$Z \equiv \sum_j e^{-\beta E_j}$
N Particles:	$Z = \frac{z^N}{N!}$
Helmholtz Free Energy:	$F = -kT \ln(Z)$
Energy:	$E = -\frac{\partial \ln(Z)}{\partial \beta}$
Pressure:	$p = \frac{1}{\beta} \frac{\partial \ln(Z)}{\partial V}$
Entropy:	$S = k (\ln(Z) + \beta E)$
Heat Capacity:	$C_V = k\beta^2 \frac{\partial^2 \ln(Z)}{\partial \beta^2}$
Alpha:	$\alpha = \frac{\partial \ln(Z)}{\partial N}$
Chemical Potential:	$\mu_j = -kT \frac{\partial \ln(Z)}{\partial N_j}$
Mean Particles Per State:	$\bar{n}_s = -\frac{1}{\beta} \frac{\partial \ln(Z)}{\partial \epsilon_s}$
Mean Square Fluctuation:	$\overline{n_s^2} = \frac{1}{\beta^2 Z} \frac{\partial^2 Z}{\partial \epsilon_s^2}$
Dispersion:	$\overline{(\Delta n_s)^2} = \frac{1}{\beta^2} \frac{\partial^2 \ln(Z)}{\partial \epsilon_s^2}$

Legendre Transformations

Energy:	$E(S, V, N) \equiv E$
Enthalpy:	$H(S, p, N) \equiv E + pV$
Helmholtz Free Energy:	$F(T, V, N) \equiv E - TS$
Gibbs Free Energy:	$G(T, p, N) \equiv E - TS + pV$

Kinetic Theory

Maxwell Distribution:	$f(\mathbf{v}) = n \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-mv^2/2kT}$
Component Dist.:	$g(v_x) = n \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} e^{-mv_x^2/2kT}$
Mean Lifetime:	$\tau = \frac{1}{n\bar{v}\sigma}$
Mean Velocity:	$\bar{v} = \sqrt{\frac{8}{\pi} \frac{kT}{m}}$
RMS Velocity:	$v_{rms} = \sqrt{3 \frac{kT}{m}}$
Most Probable Velocity:	$\tilde{v} = \sqrt{2 \frac{kT}{m}}$

Quantum Statistics

Alpha:	$\alpha = -\beta\mu$
Grand Canonical P.F.:	$Z_G = \sum_{N'} Z(N') e^{-\alpha N'}$
Photon Gas Pressure:	$p = \frac{E}{3V}$
Total Particles:	$N = \int_0^\infty f(\epsilon) \rho(\epsilon) d\epsilon$
Total Energy:	$E = \int_0^\infty \epsilon f(\epsilon) \rho(\epsilon) d\epsilon$
FD Distribution Function:	$f(\epsilon_s) = \bar{n}_s = \frac{1}{e^{\beta(\epsilon_s - \mu)} + 1}$
BE Distribution Function:	$f(\epsilon_s) = \bar{n}_s = \frac{1}{e^{\beta(\epsilon_s - \mu)} - 1}$
BEC Integral:	$\int_0^\infty \frac{x^n}{e^x - 1} dx = \Gamma(n+1)\zeta(n+1)$