Notation

$$\not{a} \equiv \gamma^\mu a_\mu \qquad \bar{\delta} f(x) \equiv f'(x) - f(x) \qquad \sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

$$\bar{\psi} \equiv \psi^\dagger \gamma^0 \qquad \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \qquad A \overleftrightarrow{\partial_0} B = A\partial_0 B - (\partial_0 A)B$$

Equations

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = 0 \qquad \mathcal{L}'(x) = \mathcal{L}(x) + \alpha \partial_\mu J^\mu(x) \qquad \frac{1}{2}(1+\gamma^5)\psi_D = \psi_W^R$$

$$Q \equiv \int \left(\frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)} \Delta\phi - J^0 \right) \, d^3x \qquad \frac{1}{2}(1-\gamma^5)\psi_D = \psi_W^L \qquad \bar{\delta} f(x) = \delta f(x) - \partial_\mu f(x) \delta x^\mu$$

Lagrangians

$$\mathcal{L}_{Free} = \bar{\psi}(x)(i\hbar c\gamma^\mu \partial_\mu - Mc^2)\psi(x)$$

$$\mathcal{L}_{QED} = \bar{\psi}(x)(i\hbar c\gamma^\mu \partial_\mu - Mc^2)\psi(x) + e\hbar c A_\mu(x)\bar{\psi}(x)\gamma^\mu\psi(x) - \frac{1}{4\mu_0}F_{\mu\nu}(x)F^{\mu\nu}(x)$$

$$\mathcal{L}_{QCD} = \bar{\psi}(x)(i\hbar c\gamma^\mu \partial_\mu - Mc^2)\psi(x) + g\hbar c\frac{\nu_a}{2}A_\mu^a(x)\bar{\psi}(x)\gamma^\mu\psi(x) - \frac{1}{4}F_{\mu\nu}^a(x)F^{a,\mu\nu}(x)$$

Dirac Matrices in Weyl/Chiral Representation

$$\gamma^0 = \begin{pmatrix} 0&0&1&0 \\ 0&0&0&1 \\ 1&0&0&0 \\ 0&1&0&0 \end{pmatrix} \qquad \gamma^1 = \begin{pmatrix} 0&0&0&1 \\ 0&0&1&0 \\ 0&-1&0&0 \\ -1&0&0&0 \end{pmatrix} \qquad \gamma^2 = \begin{pmatrix} 0&0&0&-i \\ 0&0&i&0 \\ 0&i&0&0 \\ -i&0&0&0 \end{pmatrix} \qquad \gamma^3 = \begin{pmatrix} 0&0&1&0 \\ 0&0&0&-1 \\ -1&0&0&0 \\ 0&1&0&0 \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} -1&0&0&0 \\ 0&-1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$
$$\{\gamma^5, \gamma^\mu\} = 0$$
$$(\gamma^0)^2 = (\gamma^5)^2 = -(\gamma^i)^2 = I$$
$$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$$

$$(\gamma^0)^\dagger = \gamma^0$$
$$(\gamma^i)^\dagger = -\gamma^i$$
$$(\gamma^5)^\dagger = \gamma^5$$
$$\mathrm{Tr}(\gamma^\mu) = 0$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4\eta^{\mu\nu}$$
$$\gamma^\mu \not{a} \gamma_\mu = -2\not{a}$$
$$\gamma^\mu \not{a}\not{b} \gamma_\mu = 4a \cdot b$$
$$\gamma^\mu \not{a}\not{b}\not{c} \gamma_\mu = -2\not{c}\not{b}\not{a}$$

Dirac Matrices in Dirac/Pauli Representation

$$\gamma_0 = \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&-1&0 \\ 0&0&0&-1 \end{pmatrix} \qquad \gamma_1 = \begin{pmatrix} 0&0&0&-i \\ 0&0&-i&0 \\ 0&i&0&0 \\ i&0&0&0 \end{pmatrix} \qquad \gamma_2 = \begin{pmatrix} 0&0&0&-1 \\ 0&0&1&0 \\ 0&1&0&0 \\ -1&0&0&0 \end{pmatrix} \qquad \gamma_3 = \begin{pmatrix} 0&0&-i&0 \\ 0&0&0&i \\ i&0&0&0 \\ 0&-i&0&0 \end{pmatrix} \qquad \gamma_5 = \begin{pmatrix} 0&0&-1&0 \\ 0&0&0&-1 \\ -1&0&0&0 \\ 0&-1&0&0 \end{pmatrix}$$

$$\gamma_k = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$
$$\gamma_k = \begin{pmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{pmatrix}$$
$$\gamma_5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}$$

$$\gamma_\mu^\dagger = \gamma_\mu$$
$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$$
$$\{\gamma_5, \gamma_\mu\} = 0$$

$$\gamma_5^\dagger = \gamma_5$$
$$\gamma_5^2 = 1$$
$$\sigma_{\mu\nu} = \frac{1}{2i}[\gamma_\mu, \gamma_\nu] = -i\gamma_\mu\gamma_\nu$$

Index Gymanstics

$$a^\mu \equiv \bar{a} \equiv a^\mu \hat{e}_{(\mu)}$$
$$a_\mu \equiv g_{\mu\nu} a^\nu$$
$$a^\mu b_\mu = a_\mu b^\mu$$
$$a^\mu (h_{\mu\nu} b^\nu) = (a^\mu h_{\mu\nu}) b^\nu$$
$$a_i a^i = |\mathbf{a}|^2$$
$$\delta^\mu_\nu a^\nu = a^\mu$$
$$(\hat{e}_{(k)})^i = \delta^i_{\bar{k}}$$
$$\delta_{\mu\nu} h^{\mu\nu} = h^{\mu\mu} = \mathrm{Tr}(h)$$
$$h^{(diag)}_{\mu\nu} = h_{\mu\nu} \delta^{\bar{\mu}\bar{\nu}}$$
$$(\mathbf{a} \times \mathbf{b})_i = \epsilon_{ijk} a^j b^k$$
$$\epsilon_{ijk}\epsilon^{imn} = \delta^m_j \delta^n_k - \delta^n_j \delta^m_k$$

$$\epsilon_{ijk}\epsilon^{ijm} = 2\delta^m_k$$
$$\nabla_i a^i = \boldsymbol{\nabla} \cdot \mathbf{a}$$
$$\nabla_i \nabla^i = \nabla^2$$
$$\nabla^2_{(3)} = \delta^{ij} \partial_i \partial_j$$
$$\frac{\partial a_\alpha}{\partial a_\beta} = \delta^\beta_\alpha$$
$$\frac{\partial(\partial_\mu a_\nu)}{\partial(\partial_\rho a_\sigma)} = \delta^\rho_\mu \delta^\sigma_\nu$$
$$\frac{\partial(a^\mu \partial_\mu \phi)}{\partial(\partial_\mu \phi)} = a^\mu$$
$$\frac{\partial(\partial_\mu a_\nu)}{\partial a_\sigma} = 0$$

$$\partial_\mu (a^\mu b^\nu) = (\partial_\mu a^\mu) b^\nu + a^\mu (\partial_\mu b^\nu)$$
$$\partial_\mu \phi \partial^\mu \phi = \partial_\mu (\phi \partial^\mu \phi) - \phi \Box \phi$$
$$\Lambda_\lambda{}^\kappa \Lambda^\lambda{}_\nu = \delta^\kappa_\nu$$
$$\eta_{\rho\sigma} = \Lambda^{\mu'}{}_\rho \Lambda^{\nu'}{}_\sigma \eta_{\mu'\nu'}$$
$$g_{\mu\nu} = g_{\nu\mu}$$
$$g_{\mu\nu} g^{\nu\lambda} = g_\mu{}^\lambda = \delta^\lambda_\mu$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$
$$F_{\mu\nu} = -F_{\nu\mu}$$
$$F^{0\bar{k}} = -F_{0\bar{k}} = E^{\bar{k}}$$
$$F^{\bar{i}\bar{j}} = F_{\bar{i}\bar{j}} = \epsilon^{\bar{i}\bar{j}k} B_k$$