

Physics of Climatology

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April 30, 2003

Abstract

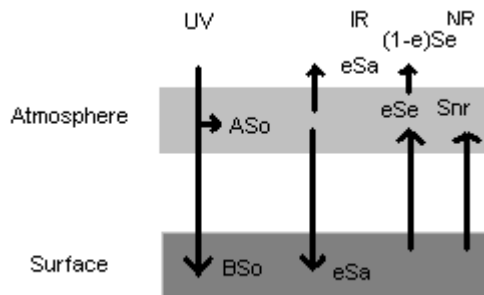
A viable simplified climate model using a three-layer representation of the earth is described. The distinct layers correspond to the earth's surface, lower atmosphere, and upper atmosphere. By considering the fact that any inequality of inward and outward energy fluxes would adjust to equilibrium according to blackbody laws, it is possible to generate layer-centric equations of climate variables. Accounting for incoming radiative fluxes in the UV spectrum and emitted radiative fluxes in the IR spectrum, most of the thermodynamics can be accounted for. With the inclusion of a parameter for non-radiative flux, a sufficient model is formed. By mapping these variables to an 864-cell grid, the computational power of Matlab can be quickly used to produce significant results. This model has the merits of being mathematically undemanding and potentially very accurate, thus justifying its appeal.

Introduction

During the course of the semester, I worked toward the development of an effective climate model. Using accepted and observed quantities, in conjunction with equilibrium and thermodynamics equations, the interrelationships of climactic variables were analyzed for consistency with the model. The procedure I took started with the establishment of a variable set, which was then followed by an investigation of the pre-established two-layer model, and then a progression into a three-layer model with two-part averaging for trends.

Two-Layer Model

The two-layer model strives to represent the thermodynamics of earth using distinct layers representing earth's surface and its atmosphere. The fluxes concerned are shown in the diagram below. Incoming radiation from the sun in the UV spectrum (S_o) is distributed to the atmosphere and the surface, with some being reflected back into outer space. The IR blackbody radiation of the atmosphere is emitted equally into outer space and onto the surface according to the product of its ideal flux (S_a) and the emissivity (ϵ – denoted e in diagram). Similarly, the IR blackbody radiation of the surface is transmitted to the atmosphere according to the product of its ideal flux (S_e) and the emissivity. The non-radiative flux (S_{nr}) caused by conduction and convection exchanges energy between the surface and the atmosphere and is taken to be upward since most regions have this characteristic.



By examining the equilibrium condition with the fluxes outlined on the above diagram, Schimdt-Helfer-Knox arrived at the following equations in “Development of an elementary climate model. II. Two-Layer cellular case”.

$$(1a) \quad 2\epsilon S_a - \epsilon S_e = AS_o + S_{nr}$$

$$(1b) \quad -\epsilon S_a + S_e = BS_o - S_{nr}$$

After acquiring the observed variables of surface temperature, surface reflectivity, and cloud cover, with the addition of global constants, the value of the remaining variables can be determined by the flux equilibrium equations. The temperature of the atmosphere is now calculated for each cell. Equations 1a and 1b are added to cancel the less-absolute non-radiative flux parameter: $AS_o + BS_o = (1-\epsilon)S_e + \epsilon S_a$, and solving for S_a yields:

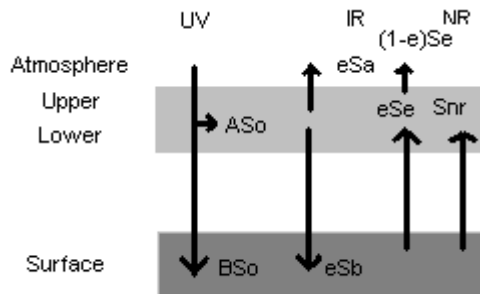
$$S_a = [AS_o + BS_o - S_e + \epsilon S_e] / \epsilon$$

Since all the parameters on the right hand side are already determined, the global average, weighted for cell areas, turns out to be 251.887 W/m^2 .

In the process of computing the two-layer parameters, a significant discrepancy was found between the standard model number of paper 1 and the calculated value for the fraction of UV solar insolation absorbed by the atmosphere (denoted A). The value calculated from observed variables and flux equilibrium equations was about half of what the standard model predicted (0.0337 vs. 0.0664). This difference could potentially propagate through other calculations in an effectual manner when this parameter is not being added to a larger value.

Three-Layer Model

The three-layer model extends the two-layer model by distinguishing between the lower and upper atmospheres. These two parts are appropriately separated because the difference in temperature caused by the proximity to the surface or outer space has a large impact on their blackbody characteristics. The basic variables from the two-layer model carry over, but the new lower atmosphere flux (Sb) is unknown and the non-radiative flux parameter (Snr) is challenged anew.



By examining the equilibrium condition with the fluxes outlined on the above diagram, Knox and Helfer arrived at the following equations in “Development of an elementary climate model. III. Three-Layer case”.

$$(1) \epsilon Sa + \epsilon Sb - \epsilon Se = ASo + Snr$$

$$(2) -\epsilon Sb + Se = BSo - Snr$$

Finding a relationship between Sa and Sb

The first objective was to estimate the flux of the lower atmosphere assuming a linear relationship to the flux of the upper atmosphere and a value of 102 W/m² for Snr. To accomplish this, equation (2) was solved for Snr, and Sb was replaced by the assumed linear function, with c as the constant of proportionality (Sb=c*Sa):

$$Snr = BSo + \epsilon cSa - Se$$

Then a binary search was executed on c until Snr was 102. The c value needed to obtain this result was 7/6. Therefore, it appears that making the assumption that the flux out of the lower atmosphere is 7/6 of the flux out of the upper atmosphere is consistent with the three-layer model. However, this is merely an estimation as there is nothing in physics to allow us to conclude such a strong statement as a linear relationship.

Calculating c (Sb/Sa)

This section provides methods for calculating the ratio of flux out of the bottom of the atmosphere (Sb) to flux out of the top of the atmosphere (Sa), which is labeled c. The relevant equations are listed below.

$$\begin{aligned}
(1) \text{ Snr} &= \epsilon c S_a + B S_o - S_e \\
(2) S_a &= [(A+B)S_o - (1-\epsilon)S_e] / \epsilon \\
(3) \text{ Snr} &= c[(A+B)S_o - (1-\epsilon)S_e] + B S_o - S_e \\
(4) c &= [\text{Snr} - B S_o + S_e] / [(A+B)S_o - (1-\epsilon)S_e]
\end{aligned}$$

The first two equations come from version (b) of the three-layer model. The third results from substituting (2) into (1). The fourth comes by solving (3) for the variable c .

The general strategy is to assume a value of Snr and solve equation (4). All the other variables on the right hand side of this formula have been calculated in keeping with the standard model. After the value for c is found, the implied value of S_b is calculated (see Appendix 3) and compared with the expected value of 360 W/m^2 . This value comes from the fact that the observed flux of 324 W/m^2 will be the ideal radiative flux times the emissivity $\epsilon=0.9$, so the value sought for the ideal flux S_b will be $324/0.9=360$. To proceed, the value of Snr is taken to be 102.

Method 1: Using Global Averages. This method computes a globally constant c with only values that are globally averaged i.e. there are no cell dependent variables thus the calculation can be done on any calculator with merely the inputs given in Appendix 1. A straightforward evaluation of (4) with averaging of the form: $c = [\text{Snr} - \langle B \rangle \langle S_o \rangle + \langle S_e \rangle] / [(\langle A \rangle + \langle B \rangle) \langle S_o \rangle - (1-\epsilon) \langle S_e \rangle]$ yields a value for c of 1.3519.

Method 2a: Using cell data – c is a global constant. This method utilizes the cell dependent variables to calculate c , then averages the cell dependent c values to find a globally constant c . Using equation (4), a matrix of c values was formed by entering this equation into Matlab. Then a weighted global average of these values was taken. (See Appendix 2). The result was $\langle c \rangle = 1.2867$.

Method 2b: Using cell data – c is a function of cell. This method utilizes the cell dependent variables to calculate c , and then keeps a matrix for c as a cell dependent variable for further computations. This method is the same as method 2a, except it omits the averaging at the end, preventing c from taking on a single value, instead it's a cell dependent matrix so S_b is determined by $S_b = \langle c \rangle S_a$ i.e. the 2nd line in Appendix 2 is replaced by $c = c_matrix$.

To test the correctness of the value for c , the implied value for S_b was compared to the formerly predicted value of 360 Watts/m^2 . By Calculation in Appendix 3:

Method 1:	$S_b = 319.144$	11.3% difference
Method 2a:	$S_b = 303.762$	15.6% difference
Method 2b:	$S_b = 275.147$	23.6% difference

Therefore, Method 1 appears to be the most promising. This is surprising because the methods that retain the most data, and hence would be expected to be the most accurate, turn out to be further off.

Appendix 1:

$A = 0.0337$ $B = 0.6649$ $\epsilon = 0.9$

So = 341.387 Se = 394.122 Snr = 102

Appendix 2:

```
c_matrix=(102-B.*So+Se)./(A.*So+B.*So-Se+IR.*Se)            %calculate c values  
c=sum(sum(c_matrix.*areas))                                    %weighted global average
```

Appendix 3:

```
Sb_calc=c.*Sa                                                    %calculate Sb values with Sa matrix  
sum(sum(Sb_calc.*areas))                                        %weighted global average
```

Determining Snr

This section demonstrates three methods for calculating the amount of non-radiative flux of heat upward from the Earth's surface. The calculation uses the shape of the non-radiative flux curve over the latitudes as a base curve and the measured value of 324 W/m² for the flux out of the lower atmosphere. Since the observed flux will be the ideal radiative flux times the emissivity $\epsilon=0.9$, the value sought for the ideal flux S_b will be $324/0.9=360$. The relevant equations are listed below.

- (1) $Snr([lat])=Snr_0(1+2\cos(2[lat]))$
- (2) $c = (Snr - B*So + Se)/(A*So + B*So - Se + IR*Se)$

The first equation expresses a latitude dependent relation for Snr that was found in Paper 2. Here it is used to form a base curve that will be scaled by the variable Snr_0 . The second equation comes from solving for the ratio S_b to S_a , as seen in 'Calculating c'. This equation will be used to test the values of Snr against the value of S_b .

The general strategy is to start with a reasonable value of Snr and to substitute it into (2) to get the ratio of S_b to S_a . Then by multiplying S_a cell-by-cell with c, a matrix of S_b values emerges, which can then be averaged to test its deviation from 360. However, Snr will not always be a simple constant, it will be a matrix determined by (1) and the variable that is being tested: Snr_0 .

Method 1: Snr as a cell variable. To perform this calculation, the Snr term in (2) is replaced with Snr_0*Snr_g , where Snr_g is a latitude dependent matrix determined from (1) (see Appendix 4 for creation of Snr_g). Then a binary search is executed manually by changing the value of Snr_0 and running through the commands in Appendix 5 to get an implied S_b average from the c_matrix average, which can be represented as solving $\langle S_b \rangle = \langle \langle c_matrix \rangle S_a \rangle$ where $\langle S_b \rangle$ is equal to the target of 360. Eventually a value near 360 is found that is close enough to yield three decimal places of accuracy on the target value of S_b . The corresponding Snr_0 value was found to be 112.601, which gives a value of 1.525 for c, and a value of 200.136 for Snr.

Method 2: Snr as a global constant. This method differs from method 1 in that the quantity Snr_0*Snr_g is weighted averaged before being inserted into equation (2). The equation for this method is identical with that of method 2a in 'Calculating c', but the result is different since now Snr is a variable and the target is the S_b value. This procedure throws away information about the shape of the Snr curve before it is used, effectively removing the need for equation (1). Despite this loss, the results turn out closer to what is expected. After the binary search, the needed Snr_0 value to get 360 is

found to be 84.466, giving $c=1.525$, and $Snr=150.142$. Note that c must be the same as in method 1 because they both end up being the constant multiplied to S_a to be tested. However in this case the intermediate c_matrix is just a matrix with all cells equal to 1.525 since the only variable in its expression has already been averaged.

Method 3: c is a cell variable (and Snr is a cell variable). This method sacrifices some simplicity in the model by making the already patchwork ' c ' into a full-blown cell dependent variable. The difference is omitting the averaging of c_matrix , and proceeding right into computing S_b from a matrix of c values as in solving $\langle S_b \rangle = \langle c_matrix * S_a \rangle$ for $\langle S_b \rangle$ equal to the target of 360. As would be expected, due to the removal of the possibility of averaging skew, the results are closer to what is expected than method 1, though still further off than method 2. After the binary search, the needed Snr_0 value to get 360 is found to be 103.245, giving $c=1.463$, and $Snr=183.523$.

These can be compared with the expected value of Snr : 102, yielding a value of 57.355 for Snr_0 (See figure 2 for comparison). The value of Snr_0 originally used to generate (1) was 40. The significant difference is a recognized discrepancy between the standard model and calculated data. However, the fact that Snr_0 is much larger than 57.355 is unexpected and may be partially attributable to averaging skew on the c matrix. This is plausible because of the anomalous character of the values, such as near the South Pole (see Figure 1).

Appendix 4: Creating Snrg

```
for k = 1:1:24
Snrl(k)=1+2*cos(pi*(k*7.5-90-3.75)/90)    %the 3.75 is to get the middle of the cell
end
for k = 1:1:36
Snrg(:,k) = Snrl    %copy the column throughout the globe
End
```

Appendix 5: Finding Sb

```
c=sum(sum(c_matrix.*areas))
Sb=c.*Sa
sum(sum(Sb.*areas))
```

Two Part Averages

As a part of the three-layer model project, I worked on attaining input data that is averaged over two 12-year periods rather than over the entire 24 years. This data would allow for the observation of global trends of climate variables in application to the issue of global warming. I wrote a java program to automatically download 12 months worth of data and average it. (See final page) However, it requires the URL of the file as a parameter, but NASA's site does not have the files independently stored. A form on the NASA web page can be used to extract the necessary files, but there is no way to access them directly without the form. Due to the complexity of this approach, I switched to the MSU dataset that was already downloaded. For this set, I used excel to first average each year's data. After this I averaged the first 12 years and the second 12 years.

Conclusion

The three-layer model continues its development and appears to be quite promising. It has suggested to us the necessity of the non-radiative flux parameter since some results gave larger values than we initially expected. There are many possible insights the model may reveal upon further analysis.

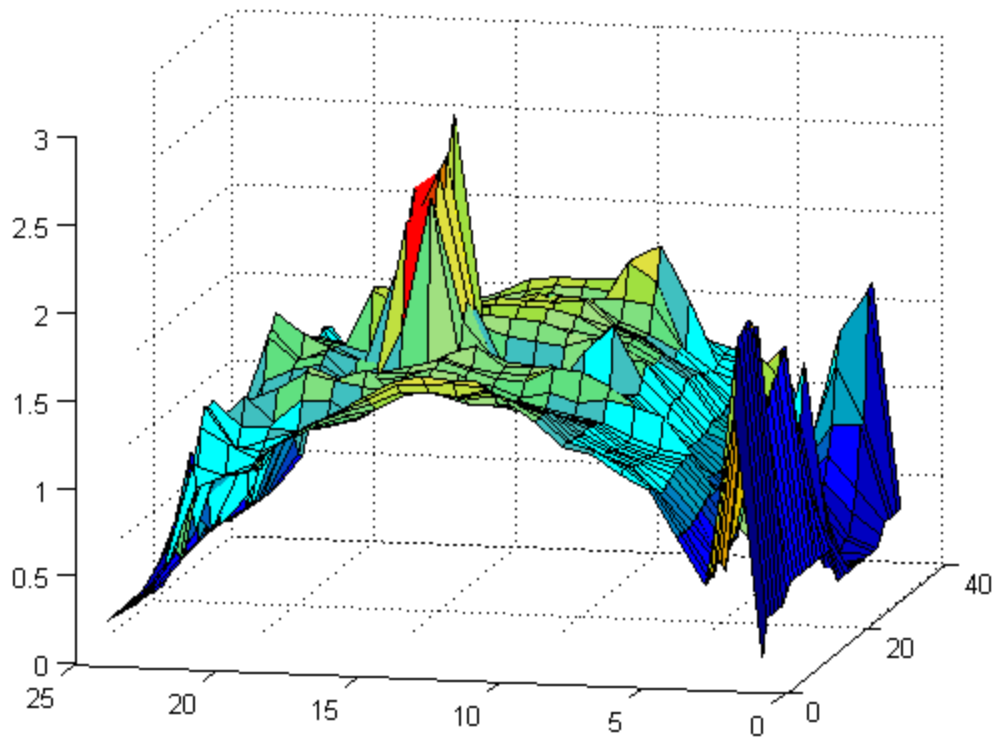


Image 1
c_matrix for method 1 of 'Determining Snr' section

The left side corresponds to the North Pole and the right side corresponds to the South Pole. There are anomalous values near the center of the grid and the South Pole that may increase the average sufficiently so that the rest of the globe has inflated Sb values.

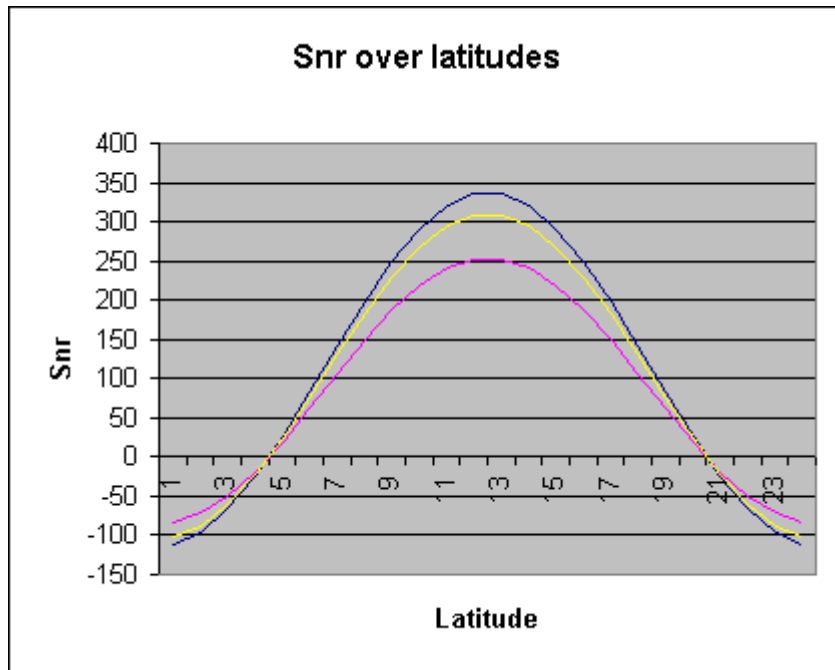


Figure 2
Snr curves over the latitudes from 'Determining Snr' Section
Blue: Method 1, Red: Method 2, Yellow: Method 3

The Snr curve is defined as $Snr_0[1+2\cos(2*(latitude))]$ and therefore is the same for all longitudes. The equation from Schmidt-Helfer-Knox originally included a term of $0.25\sin(6*(latitude))$ in the brackets, but the impact on the results for this was negligible (about 1%, and further from expected Snr) and so was omitted for simplicity.